**The Decline of the Traditional Church Choir: A Mathematical Footnote**

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## Introduction

In the first of these three articles, an account was given of the way in which the data relating to the total number of choirboys in Anglican parish-church choirs in the UK could be analysed using the theory of the logistic function. The results were then used to calculate the number of men presently in the general population who, as a consequence of the failure of the Church to nurture the traditional all-male choir, have been discouraged from singing in parish churches. This number currently stands at over a million men and is steadily rising. It was suggested in the second article that with the increasing secularization of the State education system, these men will have been denied any experience of Christianity whatever.

In an attempt to develop a better understanding of the causes of the decline in the traditional choir, an outline was presented of the philosophy underpinning the scientific method, and of how this philosophy might be applied in the present case. To this end, two hypotheses were formulated. The first relates to the observation that the total number of choirboys has, as a matter of fact, decreased dramatically over the last forty years; the second relates to the fact that the total number of girls steadily increased at the beginning of this period of change. The two hypotheses are not independent. In essence, the first is based on the idea that boys are reluctant to sing with girls whereas the second asserts that girls are naturally attracted to those activities traditionally associated with boys, in particular, the church choir. It was shown that because the two hypotheses are therefore interdependent, the decline in the number of boys and the rise in the number of girls are not only similar logistic processes but are likely to have numerically equal rate constants. This equality is the consequence of setting the notionally steady number of girls reached after a relatively long time equal to the steady number of boys that is assumed to have existed before the admission of girls.

Admittedly, these steady-state conditions must be regarded as idealizations. However, the reason for adopting them is based on one simple observation; the changes that have occurred in the number and composition of parish-church choirs have been significantly more rapid than those of the conditions in which such changes have occurred. Moreover, such changes in the conditions that have occurred may well have affected the sexes similarly. Obvious examples are the number of competent church organists and choirmasters, as well as the increase in family breakdown. Nonetheless, the number of Anglican parish churches, and hence the potential number of choirs, has remained more or less constant throughout the period under consideration, as has also the pool of potential choristers. In a philosophical vein, it should be mentioned that the analysis depends on the hitherto tacit assumption that only the total numbers of choristers need be considered, and that detailed information concerning individual choirs and the conditions in which they operate can be ignored. (The justification for this approach is to be found in the literature of mathematics under such headings as the central limit theorem [14] and the theory of errors [15]. In short, the average of a number of readings of almost any randomly-distributed quantity tends itself to be distributed according to the ‘Normal’ or ‘Gaussian’ distribution.)

**The third hypothesis**

A deficiency of the theory developed in the first two articles is that no provision was made for the possibility that having been admitted into what had been an all-male domain, girls might be reluctant to continue as members of choirs when there are few, if any, remaining boys (or perhaps even men). In other words, if the steady state mentioned above (when equal numbers of girls are continually joining and leaving choirs) is never actually achieved, what change must be made to the theory, and how will the results deduced from it be affected?

It is the purpose of this third article to provide an answer to this question. To do so, we propose a *third hypothesis*, *viz*., that in addition to the factors affecting the rate of increase in the total number of girl choristers, there is a third factor that *depends only on the prevailing number of boys*, denoted in what follows by Φ. We assume (a) that when there are many boys, this third factor has *no effect* on the rate of change of the number of girls. We also assume (b) that as the number of boys decreases, a time is reached at which there is no further increase in the number of girls and that subsequently the number of girls begins to fall. It is also probable that once this latter stage has been reached, and the number of girls begins to fall, the mathematical expression describing this last stage will be similar to, if not identical with, the falling logistic associated with the decline in the number of boys. In a philosophical sense, we should expect Occam's Razor [16] to apply in this circumstance so that (c) the decay curve for the number of girls will actually tend to become identical to that of the boys. A further application of the Razor demands that whatever analytical form we adopt for the third factor, it should be the simplest that can be found which simultaneously satisfies (a), (b) and (c).

In addition to these points, there is a further consideration that must be taken into account. Although it is required that the factor Φ should ideally be a function *only* of n(t), the total number of boys at any given time, because of the dearth of available data we shall have to assume a date at which the number of girls is a maximum. This date, regarded as an unknown parameter, can be determined approximately by inspecting the shape of the computed curve of the total number of girls plotted against time. Too early a date chosen for the maximum will result in too small a computed value for the current (2005) number girls; too late a date will result in too large a computed value. The mathematical details of the formulation of the expression for Φ (eq.27) and the solution of the resulting rate equation for the number g(t) of girls at any time (eqs.31 and 32) are outlined in the Appendix. Plots of g *vs* year are shown in figure 6 for the unmodified theory and for three particular dates (1981, 1985 and 1989) at which the number of girls reaches a maximum.

**Choice of date for the maximum number of girls**

As will be seen from figure 6, the choice of 1985 as the date for the maximum number of girls would seem to be satisfactory both in view of the above considerations and also in terms of the collective experience of a number of observers. Given that this date *per se* is only a reflection of the actual number of choirboys that existed at the time, denoted by N, and that it is this latter number that is directly involved in determining the rate of change in the number of girls, it is clear that the factor Φ depends on both the number of boys *at any given time*, as well as on the number of boys *at the time of the maximum number of girls*. Since there is a one-to-one relation between the number of boys and the corresponding date, we can regard n(t) as an *alternative measure of time*; in particular, N defines the year in which the total number of girls reaches a maximum. It then follows that the factor Φ can be expressed as a function of n and N, i.e., Φ = Φ(n, N). The equation expressing the rate of change of the number of girls at any given time (eq.17 of the second article), is then modified by the inclusion of the factor Φ, as shown in the Appendix.

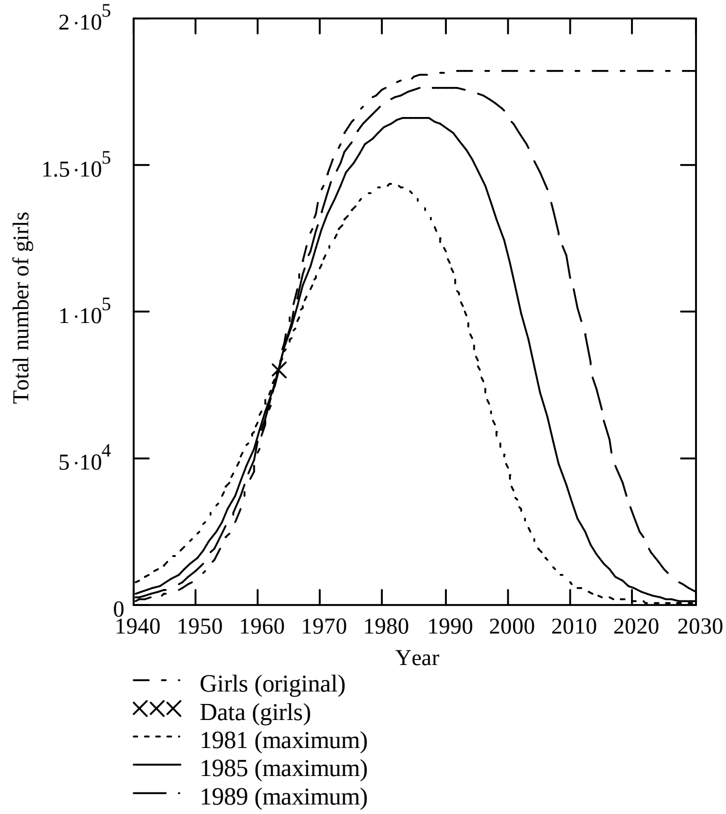


*Figure 6. Implementation of the third hypothesis, i.e. that girls are reluctant to continue singing in parish-church choirs when there are few if any boys. The rate equation leading to the logistic curve (the 'original' plot) for girls ($dg/dt = \kappa g[1-g/g_0]$) is modified to accommodate the requirement that unless there is a significant change in the prevailing conditions, the total number of girls must pass through a maximum at some (unknown) year before decreasing monotonically in a similar way to that of the logistic curve for the total number of boys. It is proposed that the latter process 'forces' the fall in the number of girls. A further necessary requirement of the modified rate equation is that both it, and its solution, should revert to their original unmodified forms when the maximum number of girls is taken to occur in the very distant future. Each of the modified logistic curves passes through the single data point representing 80,000 girls in 1963.*

**Age distributions of men and women in church choirs**

In the second article, the original expressions for the total numbers of boys and girls were used to calculate the ratio of men to women in adult parish-church choirs as well as the mean age and age range of each. It is now a straightforward matter to recalculate these results using the modified expression for the total number of girls so as to investigate the effect of the third hypothesis. The results are shown in figures 7 and 8. It will be noticed from the former that for the case in which the number of girls is assumed to reach a maximum in 1985, the effect between about 1970 and 2000 is negligible; there would have been rather more men than women throughout most of this period, both for the original calculation and also taking into account the third hypothesis. It will also be noticed that whereas at the beginning of the period covered by the calculations, the very large excess of men according to the original picture has been reduced on including the third hypothesis. This effect is a direct result of the increase in the number of girls associated with the fact that Φ for 1985 falls short of unity when n is still fairly large; to this extent, we may regard the effect as an artefact of the theory rather than an indication of a real occurrence. However, the smaller increase in the relative number of women predicted for the years after 2000 is probably an accurate reflection of the fall in the number of girls from 1985 onwards.

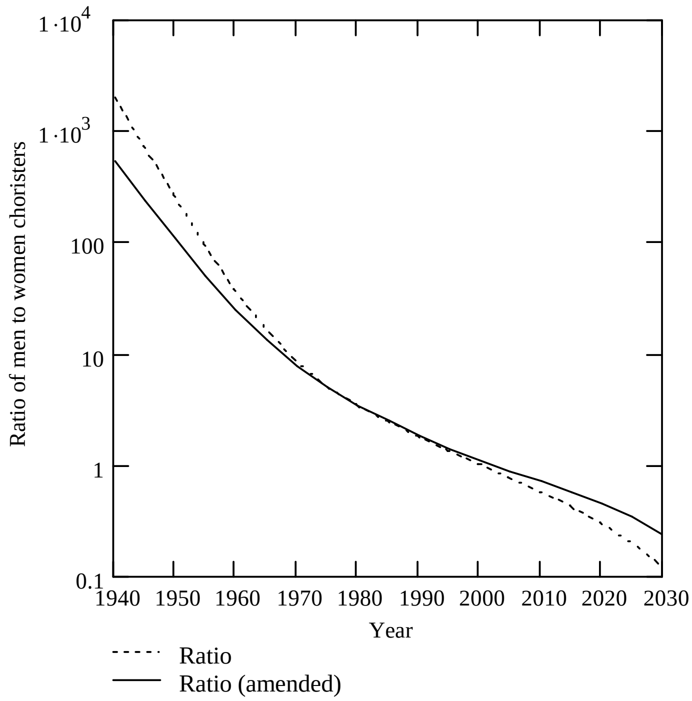


*Figure 7. The effect of introducing the third hypothesis on the ratio as a function of year of men to women in Anglican parish-church choirs. The effect will be seen as lowering the ratio (and therefore increasing the proportion of women) in the early years but slightly increasing the proportion of men in the final years under consideration.*

In much the same way, the effect of the third hypothesis on the mean age and age range for women, as shown in figure 8, can be interpreted to mean that with the falling supply of girls, these age characteristics of the women will begin to approach those of the men. From figure 1 of the first article and figure 6, we note that there is roughly thirty years between the curve for boys and the falling half of the (1985) curve for girls. We should therefore expect that in about 2015 - some thirty years after 1985, when the number of girls notionally began to fall - that the age characteristics for women in choirs would begin to resemble those of the men. This trend is clearly visible in figure 8. Moreover, we also note that the effect on the calculated age characteristics of the women of introducing the third hypothesis is negligible for the years prior to the present (2005). The inference here is that had the maximum number of girls occurred after 1985, the effect would have been even less marked. Indeed, we might even surmise that as these age characteristics are so insensitive to the date of the maximum number of girls, no great change would be observed if the maximum occurred *before* 1985.

Just as the notional maximum and minimum ages of adult choristers were presented in a table in the second article, so we can now add to that table the results of including the effect of the third hypothesis. The trends identified above are evident in Table 2 below.

| Year | Ages of men | | Ages of women (original & including effect of third hypothesis) | | | |
|---|---|---|---|---|---|---|
| | Min. | Max. | Min. | Min. (3rd) | Max. | Max. (3rd) |
| Pre-1960 | 27.7 | 62.3 | 15.0 | 15.1 | 24.6 | 27.4 |
| 2000 | 44.3 | 68.0 | 22.0 | 22.9 | 45.8 | 47.8 |
| 2030 | 62.6 | 73.8 | 27.3 | 45.5 | 61.6 | 67.7 |

*Table 2. The notional maximum and minimum ages of adult choristers in Anglican parish-church choirs for the period prior to the admission of girls in significant numbers, for the year 2000 when the age ranges of men and women are reckoned to have been about equal, and at the end of the choirboy era. The table includes data calculated on the basis of the (third) hypothesis, that girls are reluctant to remain in parish-church choirs in the absence of boys.*

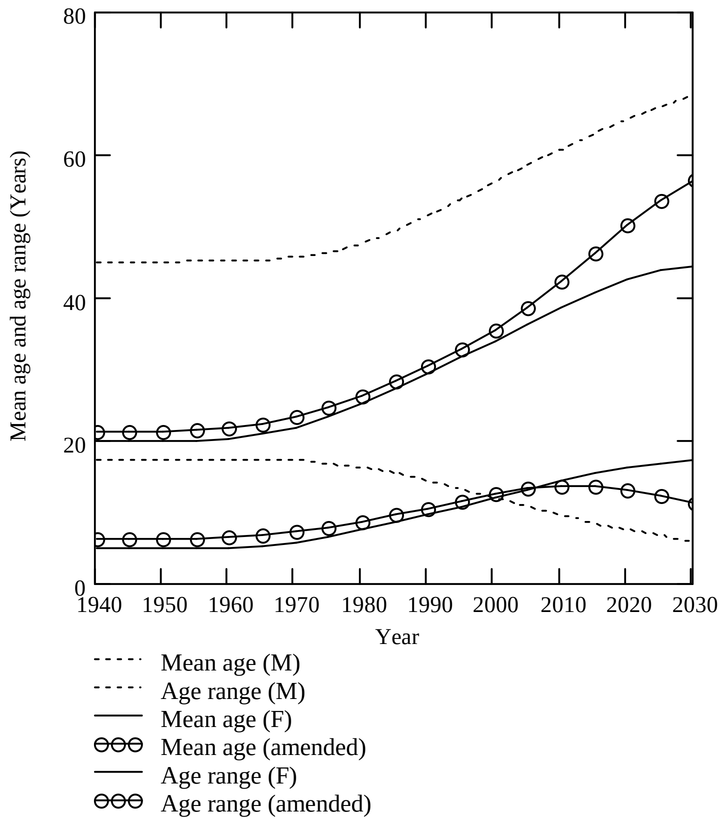


*Figure 8. The mean age and age range for men and women choristers in Anglican parish-church choirs plotted against year for the original calculation and taking account of the third hypothesis, shown to have a negligible effect until after the year 2000, when the two curves for women begin to approach the corresponding curves for men.*

## Summary and Conclusion

The original logistic theory of the decline of the traditional parish-church choir is modified to include the effect of a third hypothesis, that girls are reluctant to remain in such choirs as soon as boys begin to leave. By introducing an additional factor Φ, into the rate equation for the number g of girls, a maximum in this number can be generated so that the curve of g *vs* the year is transformed into a falling logistic similar to that for the boys. The factor Φ is a function of n and the year in which g is a maximum, expressed in the terms of the prevailing number N of boys. It is shown that if the year of the maximum is assumed to have been 1985, the effect of introducing the third hypothesis on the ratio of men to women in parish-church choirs, their mean ages and their age ranges is minimal, only becoming significant when the supply of girls begins to fall. At this time, the age characteristics of the women begin to approach those of the men. By the end of the choirboy era, in about 2030, it is predicted that there will be a total of about **seven hundred girls** in parish-church choirs, roughly the same as the current (2005) number of boys. Far from achieving the 'inclusiveness' favoured by the politically correct of the Church of England, if the assumptions made in carrying out these calculations remain valid, by 2030 there will be virtually no children singing in Anglican parish-church choirs, their places having been taken by a dwindling number of ageing adults most of whom will be women.

**References**

14. M.G.Kendall and A.Stuart, *The Advanced Theory of Statistics*, Charles Griffin & Co., London, 1958, vol.1, p.193.
15. E.T.Bell, *The Development of Mathematics*, McGraw-Hill, new York, 1940, p.538 et seq.
16. B.Russell, *History of Western Philosophy*, George Allen and Unwin, London, 1946,p.494.
17. M.Born, *Natural Philosophy of Cause and Chance*, O.U.P., 1949, p.9.

**Appendix**

Eq.17 of the second article, which can be written as

$$dg/dt = \kappa g(1 - g/g_0) \qquad (26)$$

in which the total number of girls at time t and at very long times are denoted by g and $g_0$, respectively, must now be modified to include the function Φ. As discussed in the text, this function depends on both n(t), the total number of boys at any given time, as well as the number N, the number of boys at the time of the maximum number of girls, i.e, $\Phi = \Phi(n,N)$. Eq.26 then becomes

$$dg/dt = \kappa g(1 - g/g_0)\Phi(n,N) \qquad (27)$$

In terms of the three conditions (a), (b) and (c) mentioned in the text, it is required that Φ should be unity (a) when n the number of boys is still very large, that it is zero (b) when the rate of change dg/dt of the number of girls is zero (i.e., when the maximum number of girls is reached and n = N), and that it tends to minus unity (c) as n becomes very small. The former condition 'forces' the number of girls to become stationary whereas the latter condition 'forces' eq.27 to take the form of the differential equation for a falling logistic, in fact identical in form to that for the boys, eq.13. The mathematical formulation of the third hypothesis must then be such that

$$\Phi(n,N) \to 1 \text{ when } n >> N,\ \Phi(n,N) \to 0 \text{ as } n \to N \text{ and } \Phi(n,N) \to -1 \text{ as } n \to 0.$$

These conditions are satisfied if

$$\Phi(n,N) = (n - N)/(n + N) \qquad (28)$$

a form that also satisfies Occam's 'principle of parsimony' [16] in that it is probably the simplest function that meets all the requirements; $\Phi$ is plotted against the total number of boys in figure 9. A further word of explanation regarding the forcing function $\Phi$ should be added at this point. It is clear from the analysis presented in the second article that the simultaneous solution of the rate equations for boys and girls, eqs.7 and 8, led to a common rate constant ($\kappa$) for the separate processes under the condition of equal 'saturation' numbers $n_0$ and $g_0$. It is also clear that the introduction of the forcing function $\Phi$ invalidates the simultaneous solution of these two equations.

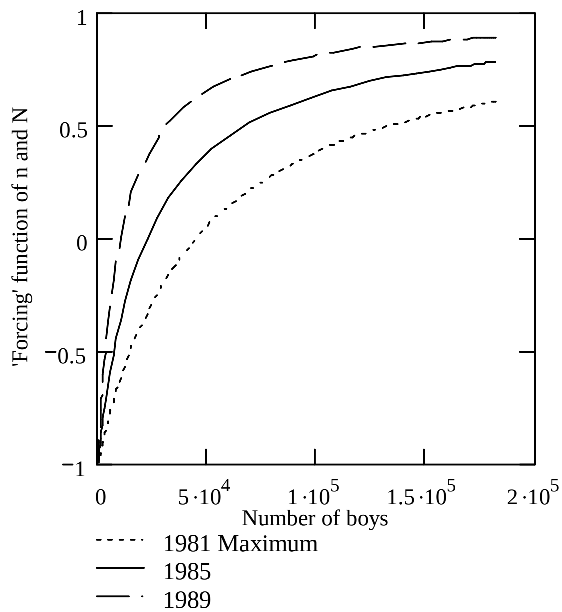


*Figure 9. Plots of the 'forcing' function $\Phi(n,N)$ (= $[n–N]/[n+N]$) vs the total number n of boy choristers in parish-church choirs; N is the number of boys at the time when the number of girls is a maximum. The role of the forcing function is to 'force' the rate of change in the number of girls to be zero in any given year (i.e. when $n = N$) so that the modified rate equation becomes $dg/dt = \kappa g[1–g/g_0]\Phi(n,N)$.*

However, we saw in the first article that, from a purely phenomenological view point, the solution to the rate equation for boys, eq.1, not only adequately fits the data but does so in accordance with known principles of the logistic process. Moreover, if we now assume that the presence of boys was, and still is the driving force behind the whole system, that it is primarily the fact that boys were involved in choirs that caused the girls to be drawn into the same activity, and that the departure of the boys will be likewise a cause of the departure of the girls, then we reach the following conclusion. If the number of girls actually passes through a maximum rather than conforming to a simple logistic process, the causal principle of antecedence [17] requires that this failure to conform cannot act retrospectively, *a cause cannot succeed an effect*. In other words, although the two rate equations have been 'decoupled' by the introduction of the third hypothesis, the solution to the equation for boys may be used to 'force' the solution for the girls to pass through a maximum. As the well-known rationale underlying the logistic growth curve for girls is still valid, so also will be the rationale underlying the introduction of the forcing function $\Phi$ provided that, if the resulting maximum in the number of girls occurs in the distant future, *the solution of the rate equation for girls reverts to the simple logistic expression* (eq.2a of the second article). Since $\Phi$ tends to unity in this circumstance, the latter condition is automatically satisfied. Eq.27 then becomes

$$dg/dt = \kappa g(1 - g/g_0)(n - N)/(n + N) \quad (29)$$

which can be readily solved when it is noted (eq.1) that $dn/dt = -\kappa n(1 - n/n_0)$ so that

$$dg/dn = -g(1 - g/g_0)(n - N)/[(n + N)n(1 - n/n_0)] \quad (30)$$

thereby eliminating the time variable. Eq.30 is easily solved (by the method of separating the variables and integrating) to obtain the result that

$$g = g_0[\{(g_0/g_1) - 1\}.\exp(S) + 1]^{-1} \quad (31)$$

in which, as before, $g_1$ denotes the number of girls in choirs in the year denoted by $t_1$ (1963), reported to have been 80,000 [2], and S is given by

$$S = \Phi(n_0,N)\ln[(n_0 - n_1)/(n_0 - n)] + [(2n_0)/(n_0 + N)]\ln[(n + N)/(n_1 + N)] + \ln(n_1/n) \quad (32)$$

in which $n_1$ denotes the number of boys in 1963, reported to have been 170,000 [2]. It should be noted that if the maximum number of girls occurs in the distant future, so that N becomes negligibly small, S reverts to the expression $\kappa(t_1 - t)$ and the equation for g becomes the simple logistic (eq.2a), as required. Plots of g *vs* time are shown in figure 6.

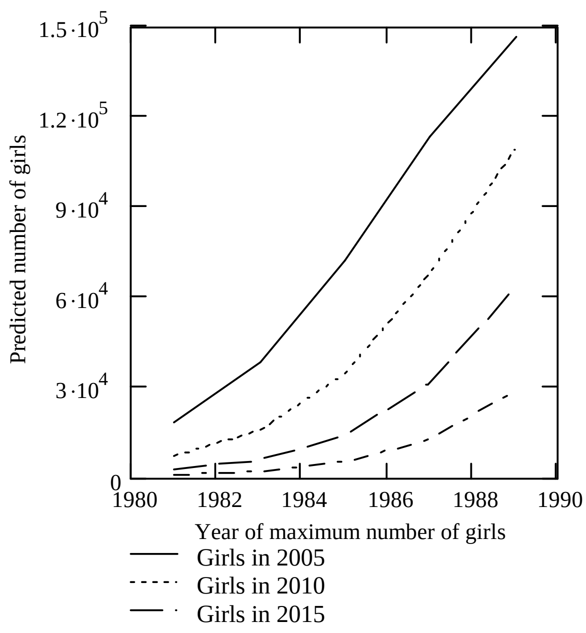


*Figure 10. The predicted number of girls in parish-church choirs calculated from the solution to the modified logistic equation; the year in which the number of girls reaches its maximum is the year in which the number of boys is N (see above). It is noted that for the maximum to have occurred in 1985, there is expected to be currently (2005) 72,300 girls, a figure that is predicted to fall to 34,800 by 2010, to 14,000 by 2015 and to 5,200 by 2020. By 2030, the figure is predicted to be* ***less than seven hundred****.*

The choice of 1985 can also be assessed by a second method which depends on calculating the number of girls for each of four different years( 2005, 2010, 2015 and 2020) and plotting them as functions of the year in which the number of girls was at its maximum. These plots are shown in figure 10.



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